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RESEARCH ARTICLE

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An additive power-transformed half-logistic model and its applications in reliability

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Abstract

An additive power-transformed half-logistic distribution is proposed to model lifetime data having bathtub-shaped hazard rate function. The model is derived as the sum of hazard rates of two independently distributed power transformed half-logistic distributions. Some properties of the model like shapes of hazard function, quantile function, mean time between failure (MTBF), probability of failure due to early birth defect and due to ageing are discussed. The graphical estimation technique based on probability plotting procedure is demonstrated to be useful in estimating the parameters of this model. The method of maximum likelihood estimation is discussed for estimating the model parameters and simulation study is considered to show the performance of the estimates. Finally the usefulness of the developed distribution is illustrated by applying it to two real life datasets from engineering reliability.

KEYWORDS

additive model, failure rate, maximum likelihood estimation, power-transformed half-logistic distribution

1 | INTRODUCTION

The situation that failure rate or hazard rate function of many mechanical and electronic components has a bathtub shape is very common. The failure rate represents the probability that a device of age t will fail in the interval $(t, t + dt)$, and this characterization is highly used in reliability engineering. A failure rate function is said to have a bathtub shape if it decreases first, then remains approximately constant and eventually increases. The bathtub-shaped function in reliability practice can be expressed as an early failure (burn-in) period, where the failure rate decreases over time, a random failure (useful life) period, where the failure rate is constant over time and a wear-out period, where the failure rate increases over time.

A significant amount of literature has been developed to model bathtub-shaped failure rates. Hjorth[1] and Mudholkar and Srivastava[2] are some of the literature where models with bathtub-shaped failure rate have been discussed. Jiang[3] developed a model with finite support to study failure rate with bathtub shape, which increases very fast in the wear-out phase. Lai and Jones[4] extended the beta density so as to make it flexible to study bathtub-shaped failure rate curves. Xie and Lai[5] and Wang[6] have developed additive Weibull and Burr models, respectively, to model bathtub-shaped failure rate. Many sufficient conditions to ensure whether a lifetime model has a bathtub-shaped failure rate or not has also been developed in the literature, for details see Glaser.[7] The additive models are another leading approach to model the bathtub-shaped hazard rate function.

This approach also leads to developing a competing risk model with two major modes of failure. The resulting hazard rates are the sum of the individual hazard rates. Recently lot of interest has been generated in developing additive

